Trigonometry

* Pythagorean theorem
  + a2+b2=c2
  + Proof
    - In a square, where sides are composed of segments with length a and b. Connect the a’s and b’s to create 4 congruent right triangles. Name its hypotenuse c. The big square’s area is a2+2ab+b2. The sum of the area of the triangles is 2ab. Remaining is the area a2+b2 and the small square inside, with area c2. a2+b2=c2
  + Triplets
    - 3 integer values that can be side lengths of triangles
* Unit circle
  + Circle with center at (0,0) and radius of 1
  + To get a unit hypotenuse triangle from any triangle, divide all sides by length of hypotenuse
  + Since the hypotenuse of a unit hypotenuse triangles is 1, using special triangle ratios, the sin, cos and tan of 0, 30, 45, 60, 90 (and other angles that use those as reference angles) can be found
    - Use line from (0,0) to (1, 0) as initial ray
    - Draw in terminal ray of angle
      * Counterclockwise for positive, clockwise for negative
    - Draw line from terminal ray’s intersection with circle to x axis, perpendicularly, forming a triangle
    - For angles greater than 90, use a reference angle
      * Find measure from terminal ray to x axis
      * Use that angle’s special triangle ratios
    - Using special triangle ratios, fill in lengths
    - Use the lengths to find sin, cos, tan
    - Y coordinate of terminal ray’s intersection with unit circle is sin, x coordinate is cos.
      * Remember x is positive in quadrants 1 and 4, negative in quadrants 2 and 3. Y is positive in quadrants 1 and 2, negative in quadrants 3 and 4.
  + Coterminal angles
    - Angles with coexisting terminal rays in unit circle
    - A multiple of 360 away from each other (60, 420, 780)
    - Has same sin, cos, tan
* Ratios
  + 30-60-90 triangle
    - Side length ratio x:x():2x
  + 45-45-90 triangle
    - Side lengths ratio x:x:x()
  + Sine (sin)
    - Sin (Θ) = cos(90 - Θ)
      * Proof
        + In triangle ABC with hypotenuse c and right angle C, A + B = 90, sin(A) = a/c and sin(B) = a/c
    - Pythagorean identity: sin2(Θ) + cos2(Θ) = 1
      * Proof
        + In triangle ABC with hypotenuse c and right angle C, sin(A) = a/c, cos(A) = b/c and cos2(A) + sin2(A) = () + () = =1
  + Cosine (cos)
    - cos(Θ) = sin (90 - Θ)
  + Tangent (tan)
    - Tan(90) is undefined
  + Cosecant (csc)
    - Reciprocal of sine
  + Secant (sec)
    - Reciprocal of cosine
  + Cotangent (cot)
    - Reciprocal of tangent
  + Arc to find angles from given ratio
* Radians
  + Way of measuring angle, aside from degrees
  + 1 radian is angle created by bending radius around circumference
  + 1 circle is 2π radians (or )
  + 2π rad = 360°,
  + 1° =rad
  + 1 rad = °
* Graphing
  + cos/sin:
  + Create a table, 1 column for x coord, one for radian measure, and one for y coordinate
    - For radian measures, use 0, , π, and 2π
    - To find x coords, take radian measure, subtract c from it, then divide difference by b.
    - To find y coords, first write the sin or cos values for the radian measures in a unit circle. Then multiply by a, then add d.
  + Plot dots and connect with smooth lines
  + Another way is to draw reference lines at midline, amplitudes and starting point. Then, draw the correct curves.
  + parent: a sin or cos(bx+c)+d
    - A is amplitude, distance from midline to max/min
    - B is frequency, amount of periods in 2π
    - D is vertical translation, shifts midline
    - D+a is max
    - D-a is min
    - is period, parent is 2π
    - - is phase start, horizontal translation
  + Tan
    - Has vertical asymptotes, as for some radian values, it’ll be finding tan of reference angle 90, which is undefined.
    - A period of parent is between 2 vertical asymptotes, length of π

Radicals and Exponents

* Roots undo exponents
  + If xn = y then y =
* Number under radical is radicand
* Number in index is the degree of root being taken
* Principal root: result from rooting is +/- if index is even, so principle refer to the +
* Simplifying radicals
  + Factor radicand into perfect square/cubes/whatevers, and leave the rest
  + Take root of the perfects and write it outside radical
  + Take negative outside radical as i
* Product rules for radicals
  + \* =
* Quotient rule for radicals
  + =
* Terms with like radicands can be added and subtracted
* Rationalizing
  + Multiply by a fraction, where denominator = numerator = (radical part of denominator)x-1 where x is the degree of the root in denominator
* Fractional exponents
  + Numerator is degree of power
  + Denominator is degree of root
* Divide exponents with same base subtracts exponent
* Multiply exponents with same base adds exponent
* Parenthesis with exponent outside multiplies exponents in parentheses (and to invisible 1s)
* Binomial with a radical term, all under a radical
  + Break it into a perfect square trinomial
* To make a negative exponent positive, make base reciprocal
* Power principle
  + If a=b, then an=bn
* Logarithms
  + logBN=E, where B is base, E is exponent, and N is BE > 0
  + Rules
    - Logxab = logxa+logxb
    - logx()=logxa-logxb
    - logxab=blogxa
    - logaa=1
    - logx1=0
    - logaax=x
    - a(=x
    - logbx=
  + Natural log: ln, aka loge
  + Common log: log, aka log10

Polynomials

* Zeros or solutions or roots are x intercepts, when y is 0
  + Fundamental theorem of algebra
    - A single variable polynomial with degree greater than 1 has at least one root
  + Linear factorization theorem
    - A polynomial has n roots, where n is its degree
  + Descartes’ Rule
    - In a polynomial, when variable is positive, the amount of sign changes is the amount of positive real roots, or that amount minus 2
    - When variable is negative, the amount of sign changes is the amount of negative real roots
  + The amount of zeros of a graph = extremas + 1
  + Factor to find roots
* Factoring
  + Used to solve roots of polynomial
  + To find rational roots, factor to the point that there is only 1 x term in each parenthesis, set parenthesis = 0 and solve for x
  + Methods
    - Factor out GCF
    - Special patterns
      * a2+2ab+b2=(a+b)2
      * a2-2ab+b2=(a-b)2
      * a2-b2=(a+b)(a-b) diff of perfect squares
      * A3+B3=(A+B)(A2-AB+B2) sum of perfect cubes
      * A3-B3=(A-B)(A2+AB+B2) diff of perfect cubes
    - Trinomial
      * Find number that adds to 2nd term and multiplies to 3rd term
    - Grouping
      * Take a few terms, put to parenthesis, then use any above methods for each parenthesis
    - Quartic
      * Highest degree is 4
      * Make y=x2, (or whatever variable is) and substitute x based on degree
      * Factor
      * Replace y to x2, based on degree (y=x2, y2=x4)
    - Using synthetic division
      * Use rational root theorem to find candidates
      * Synthetically divide polynomial by candidates
      * Find the candidate that results in a polynomial with no remainder
      * One factor is x-k, where k is candidate that worked
      * Factor quotient when polynomial is divided by x-k. Those factors are the other factors of polynomial
* FOIL
  + Expanding via distributing
* Equations can be manipulated for specific features to make working with them easier
* Standard form: in greatest to least order of degree, like terms combined
  + Degree: total of powers in a monomial
    - In a polynomial, its named by greatest degree
  + Mono 1 bi 2 tri 3 poly 4+
* Dividing
  + Long division
    - Place to standard form
    - Use 0xpower placeholder to ensure that all degrees from the greatest one to one is present
    - Divide leading term of dividend by leading leading term of divisor
    - Multiply result to divisor
    - Write it under dividend starting from right
    - Subtract and bring down next term
    - Repeat, but use the difference from prev step instead of dividend
    - After subtraction, if difference doesn’t have variable, then it’s remainder
    - Remainder over divisor, no remainder means it’s a factor
  + Synthetic division
    - Only for linear
    - Opposite of constant in divisor goes in a box
    - Use 0xpower placeholder to ensure that all degrees from the greatest one to one is present
    - Write in order the polynomial without any variables
    - Bring down leftmost number
    - Multiply it by number in box and add to next number
    - Multiply sum and add to next number and so on
    - Last number is remainder
    - To find quotient, first number is coefficient for a power of x 1 degree below degree of dividend, rest are constants for powers of x descending from the first term’s power.
    - Remainder over divisor
    - When coefficient isn’t 1
      * Solve for x in the divisor
      * Use that as the constant for synthetic division
      * Divide coefficients from synthetic by coefficient of divisor
  + Remainder theorem
    - When polynomial f(x) is divided by x-a, remainder is f(a)
    - Proof: write out the division and put quotient in quotient remainder form. Manipulate it so there’s no remainders. Apply remainder theorem concept and manipulated function would equal remainder
  + Factor theorem
    - When a polynomial is divided by another polynomial (or bi, tri), and remainder is 0, then that poly/bi/trinomial is a factor
  + Rational root theorem
    - The possible rational roots of f(x) are the numbers formed from the factors of the constant divided by the factors of the lead coefficient
      * Create a set with factors (+/-) of coefficient for each factor of lead coefficient. Divide each set by a different factor of lead coefficient.
  + Quotient remainder theorem: f(x) / b = q(x) \* b + r
  + Irrational root theorem
    - If x=a + sqrt(b) is a root then x = a - sqrt (b) is also a factor
      * (quad form is +/-)
  + Complex root theorem
    - If a + bi is a root, a - bi is also a root
* Line of best fit modeling can be done on calculator
* Transformation: when placed in a(bx-h)n+k form
  + A is vertical dilation, x reflections
  + B is horizontal dilation by reciprocal, y reflections
  + H is horizontal translation
  + K is vertical translation
  + N is polynomial’s degree, which indicate amount of roots present
* Solutions of multiple polynomials
  + Substitution, elimination, graphing for shared points/intersections
* Can be added, subtracted, multiplied, divided
* When in fractions, can be added/subtracted once denominator is make equal through multiplication of fraction that equals 1, or multiply LCD to whole equation to eliminate denominator
* When in fractions, can be multiplied/divided like any fractions
* When in fractions, to simplify, write everything as factors, and negate identical factors in numerator and denominator, (cross simplify)
* When in fractions, to solve for variable, multiple whole equation by LCD to eliminate denominator or multiply by fraction that equals 1 to make terms combinable and combine terms so cross multiply can solve for variable, or variable can be solvable by reverse order of operation
  + If a possible value of variable makes a denominator 0, it’s extraneous
  + Check with substitution to check for false statements that indicate extraneous (to be excluded because it’s not possible)
* Expanding binomial to the power of n
  + Get pascal’s triangle
  + Make a = first term in binomial, b = second term and rewrite it that way
  + Look at row n, count starts from 0
  + Write n + 1 sets of a b
  + For a, left to right, give the first one the power n, and the proceeding ones decreasing by 1
  + For b, same thing, but start at 0 and increasing
  + The coefficients are in order as seen in the row in pascal’s triangle
  + Substitute a and b back to original values

Functions

* Rule where valid input/x value/domain maps to only one output/y value/range
* Vertical lines on graph would only have one intersection with function
* Functions can be added, subtracted, multiplied, divided
* Continuous
  + All points adjacent
  + Constant slope
    - Has straight lines
    - Linear
      * Parent: f(x)=mx+b
        + M = slope
        + B = y intercept -translate vertically. Represents initial
      * Arithmetic sequence
    - Absolute value
      * Parent: f(x)=a+c
        + A dictates vertically, and may reflect graph over x (when negative), multiplied to slope
        + B translates horizontally (“x lies” -move opposite)
        + C translates vertically
        + A negative x reflect over y axis
        + Z dilates horizontally by reciprocal
  + Changing slopes
    - Has curved lines
    - Average rate of change:
    - Quadratic
      * Standard Parent: f(x)=ax2+bx+c
        + A may reflect graph over minimum, and will dilate horizontally by reciprocal
        + C translates vertically
      * Vertex form
        + f(x)=a(x-h)2+k
        + Vertex is (h, k)
        + Can be derived from completing the square
        + A = 0.25p, where p is distance from a point to focus or directrix
      * Intercept form
        + f(x) = a(x-p)(x-q)
        + P and q are roots, “x lies”
      * Focus-directrix form
        + Any point’s distance to the focus is equidistant to directrix
        + Open vertically: (x-h)2=4p(y-k)

(h, k) is vertex

P is distance from vertex to directrix or focus

If p is positive: if vertical: open up, horizontal: open right

If p is negative: if vertical: open down, horizontal: open left

* + - * + Open horizontally: (y-k)2=4p(x-h)
        + If both parenthesis are to power of 2, graph becomes circle, not conic
        + To derive: take distance from a point to focus, then set equal to distance from that point to directric
      * Axis of symmetry is vertex’s x coordinate
        + Also solvable by
      * minimum/maximum is vertex’s y coordinate
      * Solution is intersections to x axis
        + Solve for x when y = 0
      * Y intercept is when x = 0
      * Parent opens up
      * When in standard form, sum of roots is , and product of roots is (test: quadratic formula with a, b, c)
      * Solving for x when y = 0
        + Factoring

Factor out GCF, find numbers that add to middle term, and equal last term when multiplied, FOIL to check, solve each parenthesis for x.

Equations can be manipulated for specific features to make working with them easier

Special patterns

a2+2ab+b2=(a+b)2

a2-2ab+b2=(a-b)2

a2-b2=(a+b)(a-b) -difference of perfect squares

* + - * + quadratic formula

There was once a negative boy, who was so squared, he lost out of 4 awesome chicks, making him cry all over 2am

This is derived from completing the square of the parent function

B2-4ac is discriminate

0: one real solution

>0: two real solutions

If perfect square, roots will be rational, else, not

<0: one imaginary solution

* + - * + completing the square

Isolate c term, then add to both side the result of ()2. Factor non c term to be perfect square. Then, take square root of both side, then solve for x.

* + - Cubic
      * Parent: a(bx)3+c
        + A reflects over x when negative, and dilates vertically
        + b dilates horizontally by reciprocal
        + C translate vertically
      * Factoring cubic
        + Perfect cubes

A3+B3=(A+B)(A2-AB+B2) sum of perf cube

A3-B3=(A+B)(A2+AB+B2) diff of perf cube

* + - Cube root
      * Parent: a\*+d
      * a is vertical dilation
      * b is horizontal dilation
      * c is horizontal translation (x lies)
      * d is vertical translation
    - rational/reciprocal
      * Has vertical and horizontal asymptote
        + Domain and range will exclude asymptote
      * Parent: ()+c
        + As a > 1, it vertically stretches, while a < 0 vertically compresses
        + B is horizontal translation (x lies)
        + C is vertical translation
        + When a is negative, rotate 90o
    - Square root
      * Parent: d\*+b
        + D is vertical dilation
        + C is horizontal dilation
        + A is horizontal translation (x lies)
        + B is vertical translation
    - Exponential
      * Parent: f(x)=abcx+d+e
        + A dictates y intercept, dilates vertically (y axis). Indicates initial
        + d translates horizontally (“x lies”)
        + e translates vertically
        + C dilate horizontally (x axis) by reciprocal
        + B is the growth/decay rate (>1: grow, <1: decay)
        + In real life application x is amount of iteration
        + When compounding

Divide b, and multiply x by amount of times compounding in a year

If compounding continuously use Pert

* + - * Asymptote
        + Range value that graph approaches but doesn’t reach
      * Geometric sequence
    - Logarithms - inverse of exponential
    - Sine, cosine
* Discontinuous
  + Gap between points present
  + Piecewise
    - Has inequalities, each with a different value assigned
    - Output depends on which inequality input satisfies
    - Step
      * f(x)=+b
      * A translates horizontally (“x lies”)
      * B translates vertically
      * Start full circle at one point, 1 x unit later, empty ending circle. Up one unit above ending circle, it starts again.
  + Tangent, cotangent, secant, cosecant
* Symmetrical
  + Quadratic
  + Absolute value
  + Widens when x2 term (quad) or x term (abs) decreases
  + Rational
  + Cube root
* Non symmetrical
  + Exponential
  + Linear
  + Piecewise
  + Step
  + Cubic
  + Square root
* Even
  + Has y axis a line of symmetry
  + f(x) = f(-x)
* Odd
  + Symmetric through origin
  + -f(x) = f(-x)
* Models
  + Scatter plot and make line of best fit (close to points) to predict values
  + Residuals (r) indicate distance from line of best fit to actual points
  + Coefficient of determination (r2)
    - Between 0 and 1
    - Closer to 1: model is good
* Composite
  + The output of a function is the input of another function
  + Indicated as (f(g(x)) or (f\*g)(x)
  + Function letter closest to x will have its output used the input of the function letter left of it
* Inverse
  + Functions are inverse when domain of one is range of the other, range of other is domain of one, and so on (g(f(x))=x, (f(g(x))=x so g-1(x)=f(x)
  + Reflects over y=x
  + Indicated with -1
  + When composite, output = input
  + To find inverse, switch domain (x) and range (y) and put into y= form
* Transformation of f(x)
  + a(bx+c)+d
  + A: vertical dilations
  + B: horizontal dilation by reciprocal
  + C: horizontal translation by opposite
  + D: vertical translation
* Odd degree functions
  + If leading coefficient is >0, function approaches negative infinity as x approaches negative infinity, and approaches positive infinity as x approaches positive infinity
  + If leading coefficient is <0, function approaches positive infinity as x approaches negative infinity, and approaches negative infinity as x approaches positive infinity
* Even degree functions
  + If leading coefficient is >0, function approaches positive infinity as x approaches both approaches positive and negative infinity
  + If leading coefficient is <0, function approaches negative infinity as x approaches both approaches positive and negative infinity
* The amount of zeros of a function = extremas + 1
* A root with even multiplicity (even #) will touch x axis but won’t cross
* A root with odd multiplicity will sway through x axis at root value
* Set notation
  + ( is approach
  + [ is included
  + ( or [ lower bound, upper bound ] or )
  + Infinities can only be approached
* Other notation things
  + U is or
  + is and

Complex Numbers

* *i* = -imaginary
* Periods of i
  + i -> -1 -> -i -> 1
  + Divide exponent by 4
  + If remainder = 0: 1
  + If remainder = 1: i
  + If remainder = 2: -1
  + If remainder = 3 = -i
* Complex
  + A + bi
  + A is real while bi is imaginary
  + Complex conjugates- complex # where b*i* term turns opposite
  + (a+bi)(a-bi)=a2+b2
  + When adding/subtracting, add a terms, then bi terms
  + When multiplying, multiply the a terms then the b terms. Subtract b terms product from a terms product to get new a term. Multiply the a term of one complex number by the b term of other complex number, and add to product of the remaining a term and b term, to get new b term.
  + When dividing, write as fraction and multiply by ()

Equations and Inequalities

* Circles: (x-h)2+(y-k)2=r2
  + Vertex (h, k), r radius
* System of equations: multiple equations, multiple variables
  + To solve:
  + Graphically
    - Find points of intersection
  + Algebraically
    - Substitution
      * Isolate one variable in one equation and substitute in other equation, solve, find other variable and substitute to original
    - Elimination
      * Multiply both equations so that when the equations are added or subtracted, one term will be eliminated. Solve for remaining variable, then substitute value to original equation.
    - Can also use rref matrix
* Solving log equations
  + When logs terms are on one side
    - First, condense the log term. Then, convert to exponential form. Next, solve, and make sure to not take the log of a negative number
  + When log terms are on both sides
    - First, condense log terms. Then, If logax=logay, cancel the loga’s. Solve and check that you won’t take log of negative numbers.
  + Variable in exponent
    - First, isolate exponential term. Then, take the natural or common log of both sides. Next, isolate the variable using log rules and solve.

Sequences and Series

* Arithmetic sequence
  + linear
  + Difference between consecutive terms are common
  + Explicit
    - an=a1+d(n-1)
    - One equation
    - N is term number, an is value of nth term of sequence and d is common difference
  + Recursive
    - 2 equations, and each term is dependent on previous term
    - a1=? equation to declare first term
    - an=an-1+d
  + Series (Sn)
    - Expression from adding terms of series
    - Sigma notation to find sum of finite series
      * Sn= where n is number of elements in series
* Geometric sequence
  + Exponential
  + Difference between adjacent terms have common ratio
  + Explicit
    - an=a1rn-1where r is common ratio
  + Recursive
    - a1=?
    - an=ran-1
  + Series

Probability

* Experimental probability
  + Based on observation and collected data
* Theoretical probability
  + Not based on experiments
  + P(A)=
  + Can use a 2-way table or Venn diagram to help solve
* Sample space: collection/set of possible outcomes
* Independent: outcome of one event doesn’t affect the occurrence of another
* Dependent: outcome of one event affects the occurrence of another
* P(AB)=P(A)+P(B)-P(AB)
* If independent, P(AB)=P(A)\*P(B) is true too
* Conditional probability
  + Probability something happens given another condition is true
  + P(A|B)=
  + If independent, the following is also true
    - P(A|B)=, ∴ P(A|B)=P(A)

Statistics

* Standard deviation
  + Population
    - Based on all elements
    - Where x̄ is mean, xn is the nth member, n is the number of members and is population standard deviation
  + Sample
    - Based on subset of the population
  + Describes how spread out data points are (lower = closer)
* Mean (x̄): average
* Median: the middle value, or average of the 2 middle values
* Quartiles: resulted from dividing the data set into 4 groups
* Interquartile range: difference between Q3 and Q1
* Mode: the data point that occurs the most
* Range: difference between greatest and lowest data point
* Normal distribution: data points distributed in a way that can be represented using a normal bell curve, symmetrical at the mean
* Standard normal curve: graph with normal distribution, mean of 0 and standard deviation of 1
* Uniform distribution: equal number of data points throughout
* Skewed left: has more data points on the right
* Skewed right: has more data points on the left
* Z-score: number of standard deviations a data point is away from the mean
  + , where x is the data point, s is the standard deviation andis mean
* Empirical rule: true for normal distribution
  + 68% of the data points lie within 1 standard deviation from mean
  + 95% of the data points lie within 2 standard deviations from the mean
  + 99.7% of data points lie within 3 standard deviations from the mean
* To find the area of the data set a range covers, use normalCDF(lower, upper, mean, standard deviation)
* invNorm(area, mean, standard deviation) is for finding what value is at the given percentile
* Margin of error
  + Error of a proportion
    - Where p is the proportion and n is number of members
  + Error of a mean
    - Where S is the standard deviation and n is number of members
  + Error of the mean of multiple simulation
    - 2S
    - Where s is the standard deviation
* Confidence interval
  + From value-margin to value+margin
  + Values that fall within the range are normal
* Simulated group mean difference
  + For simulations with a population divided into 2 groups for testing special treatment and many trials of 2 re randomized groups
  + Find the difference in mean between the 2 special treatment groups
  + Graph the mean differences of the trials of the 2 re randomized groups
  + Find percentage of the re randomized groups that have a mean difference greater or less (depending on scenario) than the mean difference of the 2 special treatment groups
  + If the percentage is less than 5%, the difference between the 2 special treatment groups is significant and was caused by the special treatment and not by chance
* Data collection
  + Sample: A subset of a population being studied
    - Unbiased: a sample that’s a good representation of the population
    - Biased: a sample that’s not a good representation of the population
      * Can lead to bias when question encourages respondents to answer in a particular way
      * Bias can arise when the subject is too sensitive
      * Can lead to bias when the respondent isn’t given enough information about topic
  + Experiment
    - Purposely manipulate something to find how that creates an effect
    - Have an experimental group and a control group
  + Observational: data is collected from existing data set, not from an experiment or special treatment
  + Survey: ask for data from a population

Modeling

* Line of best fit: the function that best represents the data set
* Can get from running regressions
* R2 value closer to 1 means better representation

BTHS Semester 1: <https://drive.google.com/file/d/17Hat2MTRR4CIEojX0kD649fnmm0Eio-x/view?usp=sharing>

Other uncategorized things:

()t + ()t = 1, where a is time alone for item a, b is time alone for item b, and t is time together. Basically, and are rates for doing something, and t is rate when they work together

= + c, where d is the distance traveled, a is the speed of item a, b is speed of item b, and c is how much longer it takes item b